**JeTseT** JET Simulations, Experiments and Theories

# Numerical Simulations in MHD. Tools and Methods

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MARIE CURIE ACTIONS

**www.jetsets.org**

# Outline

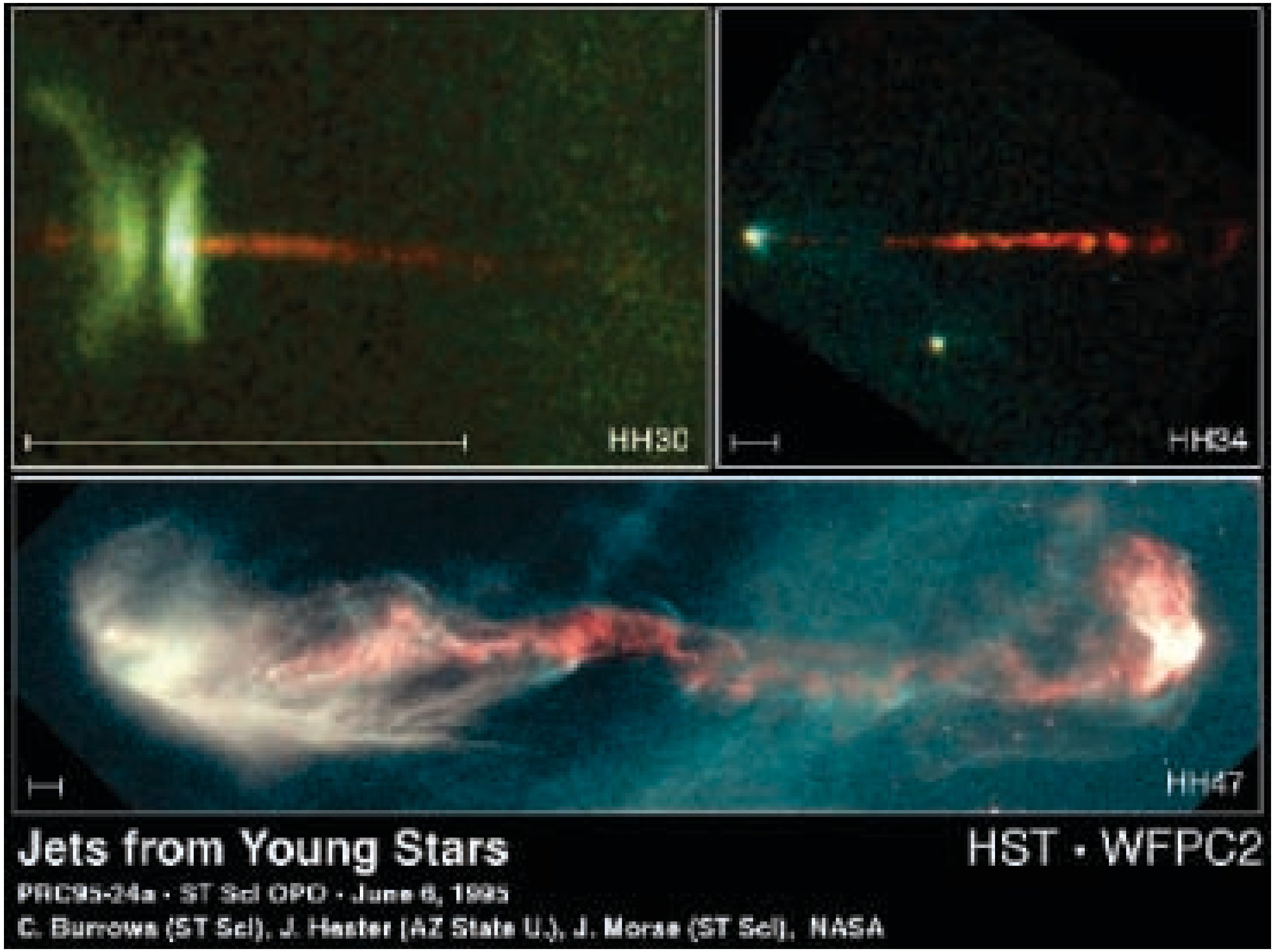
- Introduction
- Methods
- Tools
- Prospects

# Introduction 1

- **Astrophysical media**
  - multicomponent?, large gradients
  - viscosity, resistivity
  - chemistry?
- **Analytical solutions and their limitations**
  - “exact” solutions
  - analytical approximations
- **Art of numerical simulations**
  - numerical approximations
  - setup, choice of B.C. and I.C.

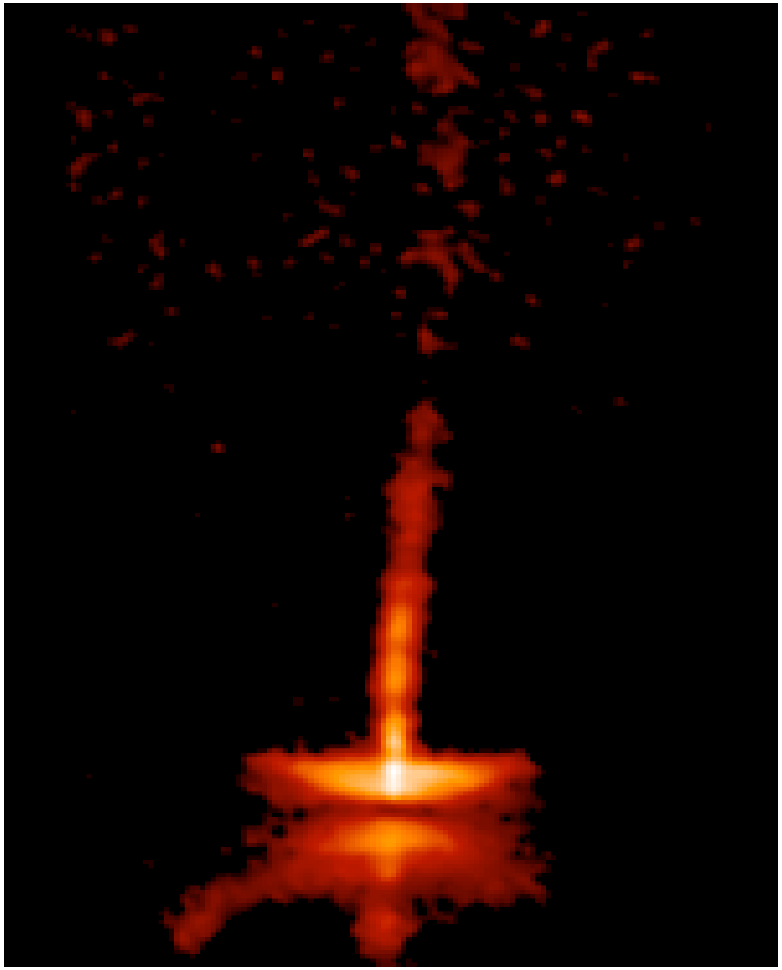
# Introduction 2

- Observations – young stellar objects



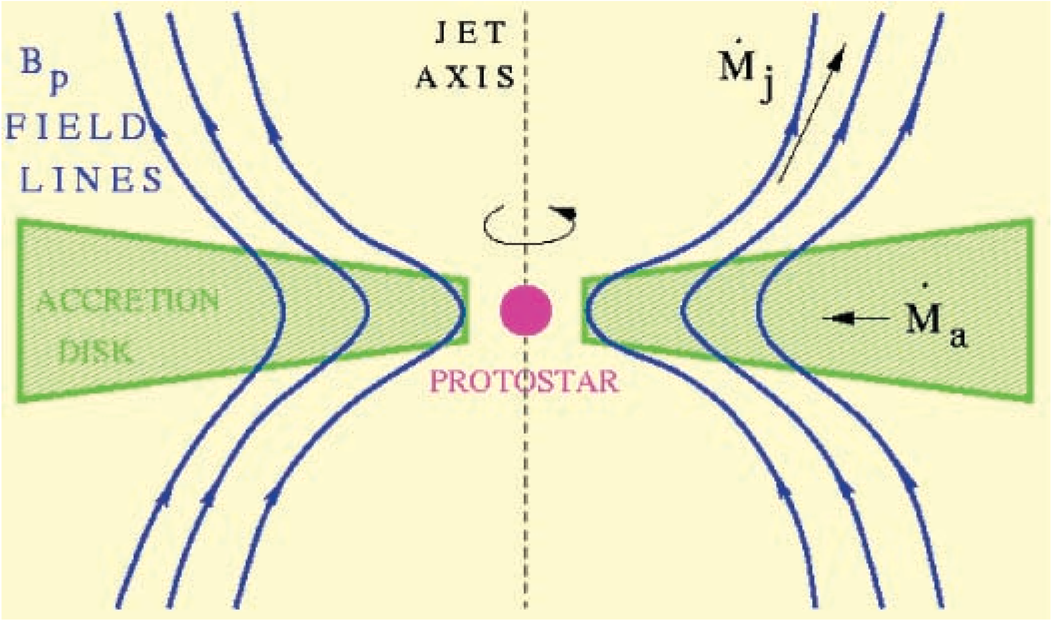
# Introduction 3

- More precise observations – HH30 1995-2000



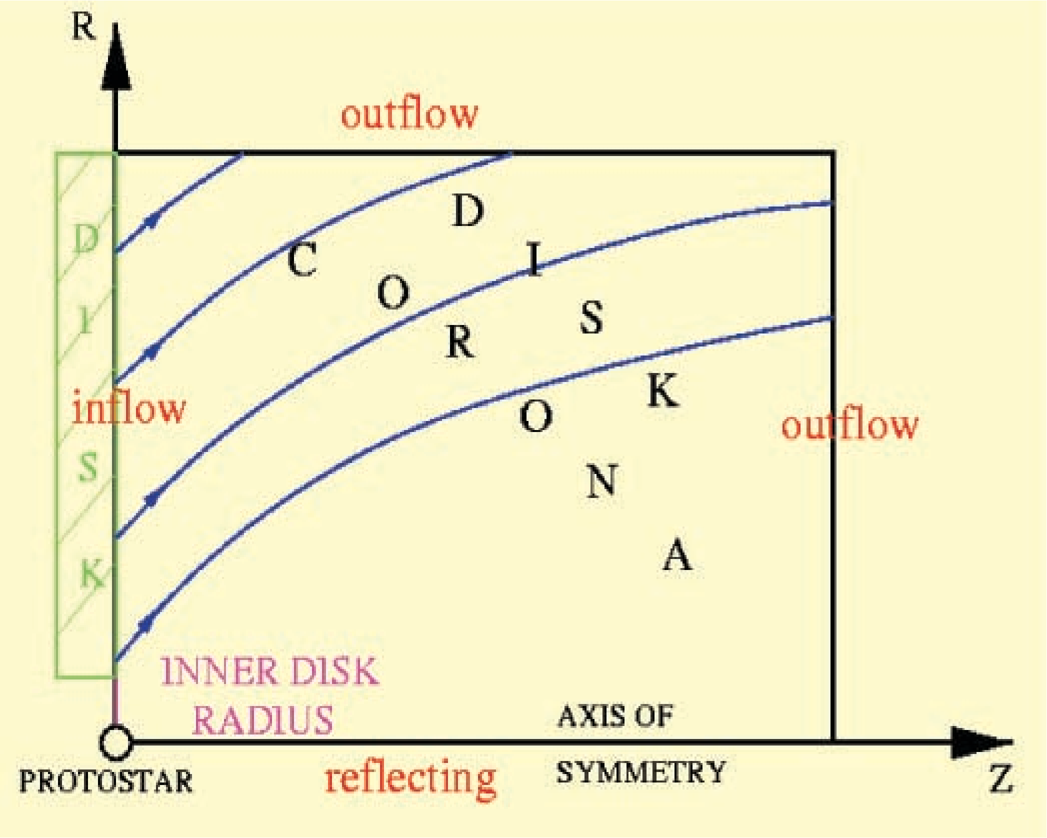
# Introduction 4-Example

- **Example: Sketch of the model**



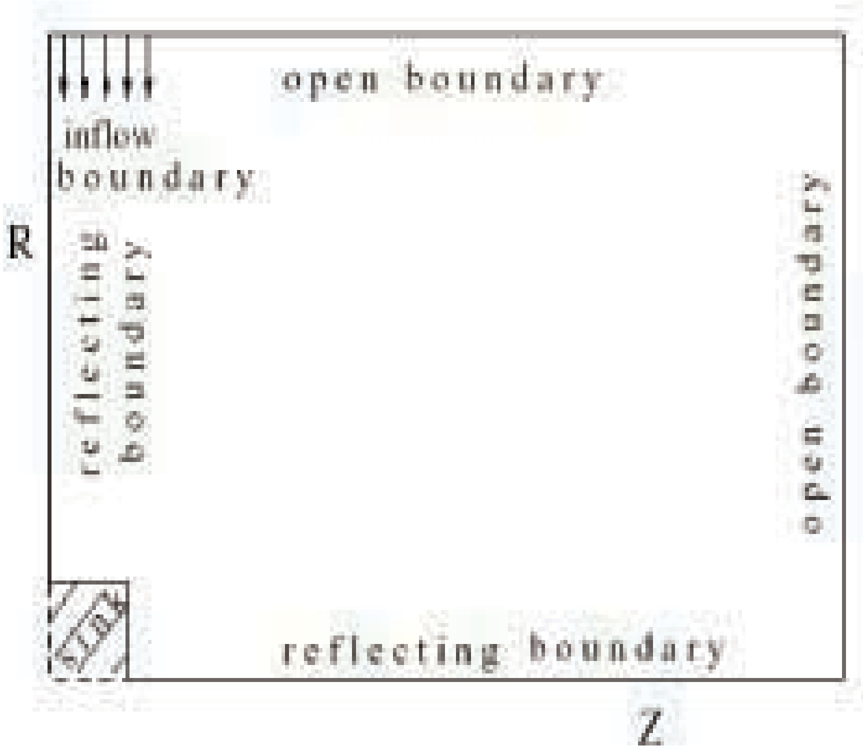
# Introduction 5-Example

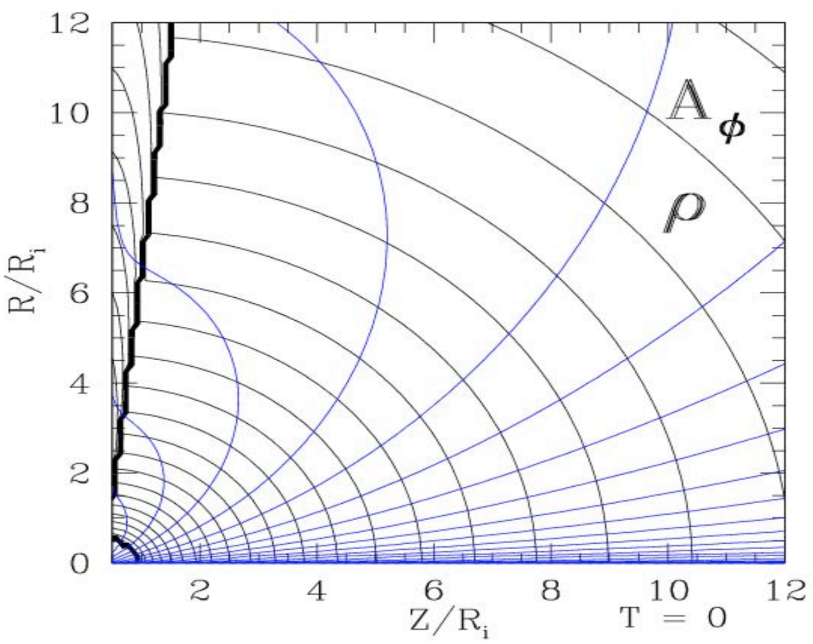
- Setup of the jet without disk included in the comp. box



# Introduction 6-Example

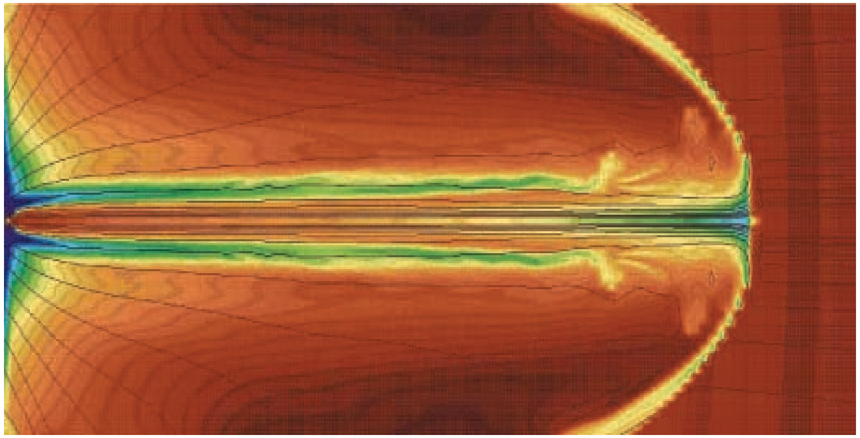
- Setup of the jet with the disk included in the comp. box

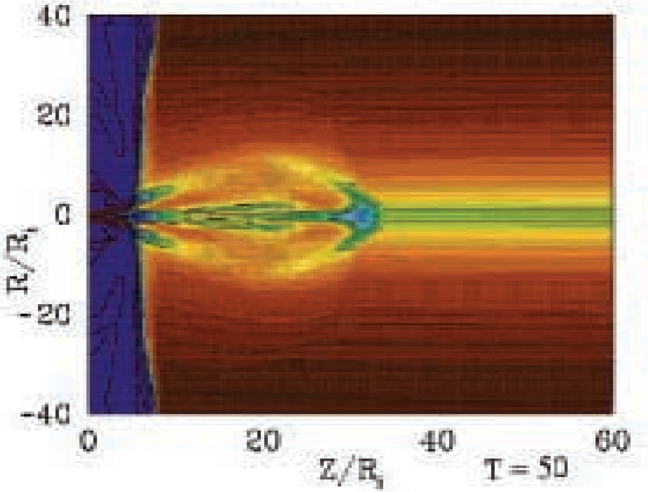


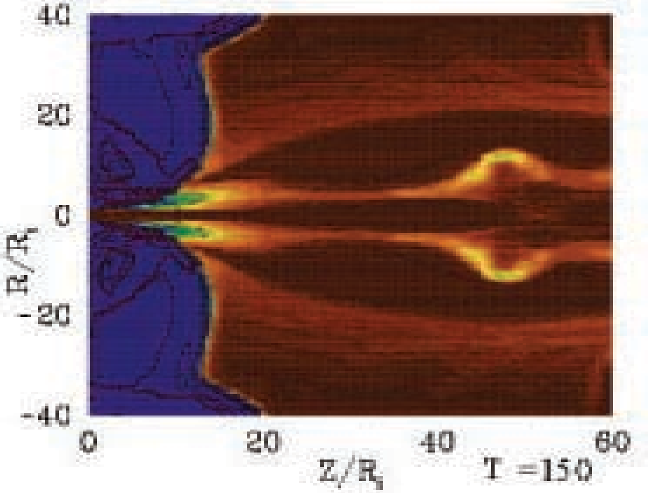



# Introduction 7-Example

- Quasistationary states





# MHD=Magnetohydrodynamics



- ZEUS-3D
- ZEUS-MP

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\rho \left[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} \right] + \nabla p - \rho \nabla \left( \frac{GM}{\sqrt{r^2 + z^2}} \right) - \frac{\mathbf{j} \times \mathbf{B}}{c} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times \left( \mathbf{u} \times \mathbf{B} - \frac{c\mathbf{j}}{\sigma} \right) = 0$$

$$\rho \left[ \frac{\partial e}{\partial t} + (\mathbf{u} \cdot \nabla)\, e \right] + p(\nabla \cdot \mathbf{u}) - \frac{\mathbf{j}^2}{\sigma} = \mathbf{0}$$

$$\nabla \cdot \mathbf{B} = \mathbf{0}$$

$$\frac{4\pi}{c}\mathbf{j} = \nabla \times \mathbf{B}$$

$$p = K\rho^{\gamma},\; e = \frac{p}{\gamma - 1},\; \gamma = \frac{5}{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \quad \eta = \frac{c^2}{4\pi\sigma}$$

# Numerical solutions

- **The set of differential equations for:**
  - ρ (density)
  - ρu (momentum)
  - ρe (energy)
  - B (magnetic field)
  - p (pressure)
- **Numerical schemes (discretizations)**
  - differ in addressing the contact and shock discontinuities in the flow
  - overall accuracy depends on the way these problems are solved
  - finite differences, finite volume and finite elements method

# Finite differences method

- **Use of neighbouring points: approximation of the derivative of an unknown quantity *U* at a grid point by the ratio of the difference in *U* at two adjacent points to the distance between the grid points.**
  - mostly on regular mesh

# Finite volume method

- **The variables are approximated by their average values in each volume, and the changes through the surfaces of each volume are approximated as a function of the variables in neighbouring volumes.**
  - on both regular and irregular mesh

# Finite elements method

- **Also splits up the spaces into small pieces (called *elements*) as in finite volume method. But now a grid point exchanges the information with all the other grid points with which it shares an element.**
  - no advantage of regular mesh

- **All this possible in AMR (adaptive mesh refinement) approach**

# Schock-capturing methods

- Classical
  - linear numerical dissipation terms
  - the same dissipation at all grid points
  - for smooth and weak-shock solutions
  - symmetric or central discretization
  - no info used about wave propagation
- *and* modern method
  - non-linear numerical dissipation
  - feedback for adjusting of dissipation
  - every cell adjusted
  - based on "upwind differences" (PDEs solutions dependent on velocity sign)
- *intermediate method*
  - linear numerical dissipation terms, non-linear switch functions

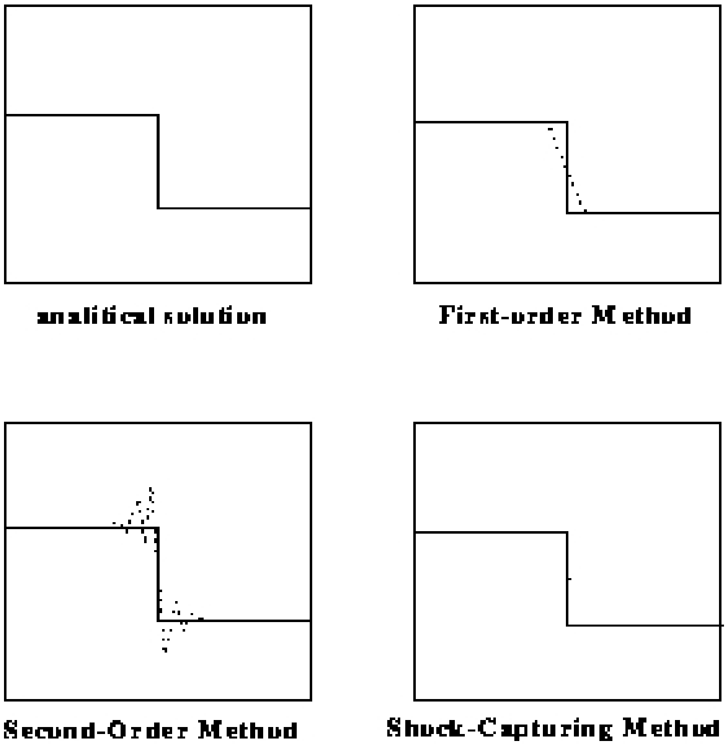
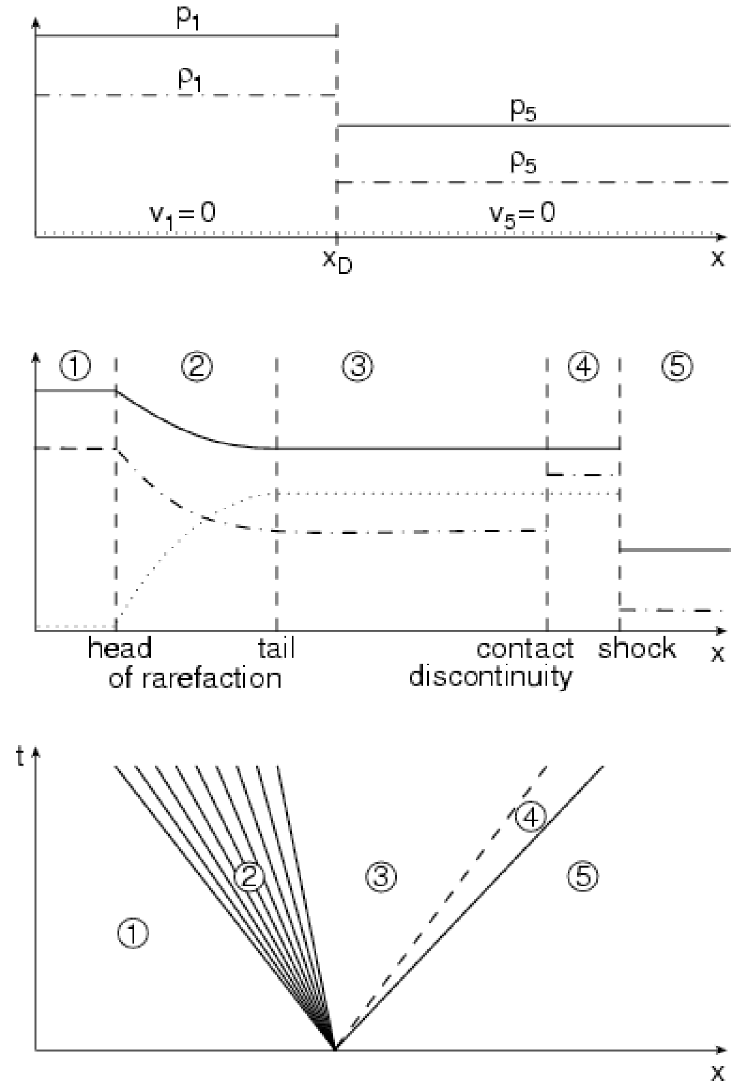



FIG. 2. *Examples of numerical discretizations*

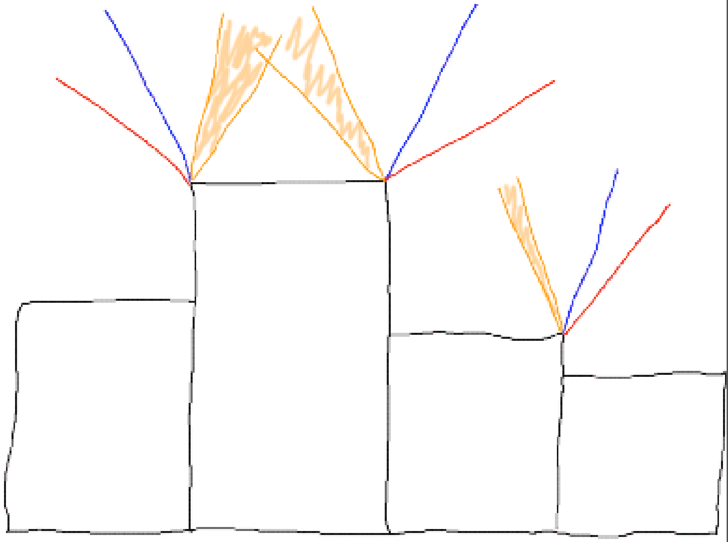
# Riemann problem

- **The simplest initial value problem**
  - discontinuous data
  - two separated constant states
  - breakup of discontinuity
  - two types of waves:
  - shocks *and* rarefactions
  - contact discontinuity (moving)



# Godunov-type schemes

- **Godunov (1959) exact Riemann solver**
  - solving of a separate Riemann problems
  - solving at each cell boundary



- **Three steps (*U*=1,v,e):**
  - reconstruction of ρU(x) from cell ρU-es
  - solving of Riemann problems for Δt
  - computing the fluxes across cell boundaries and averaging of ρU(x)-es to obtain ρU-es
- ***Useful method, but in original form too diffusive***

# Riemann solvers

- **What is usually used is combination of Godunov's concept with high-order reconstruction (solution averaging):**
  - Van Leer (1979): MUSCL (Monotone Upwind Schemes for Scalar Conservation Laws)-linear reconstruction: approximation of piecewise-linear Riemann problems by piecewise-constant Riemann problems including slope-limiter, solution of the Lagrange equations and Eulerian remapping.
  - Colella & Woodward (1984): PPM (Piecewise Parabolic Method): piecewise parabolic reconstruction via primitive functions, contact steepening.
  - Approximate (linearized) Riemann solvers may serve as well in splitting the flow into waves with different characteristic velocities and upwind directions.

# Alternative concepts

- **(Approximate) Riemann solvers account for upwinding and shock capturing but:**
  - involved computations, costly in CPU time
- **Alternatives-simpler-make use of:**
  - von Neumann-Richtmyer viscosity
  - Runge-Kutta steps
  - operator-splitting of advection and pressure terms
- **Other Riemann solvers:**
  - Approximate Riemann Solver of Roe (1981)-solves exactly a linearised problem (by an algorithm by Roe), instead of looking for an iterative solution of the exact original Riemann problem
  - Harten-Lax-van Leer-Einfeldt (or HLLE) scheme (1988)-the energy of a flow is highly kinetic

- **Directional (dimensional) splitting**
  - applying the operator splitting to the spatial derivatives in Euler eqs.

$$\frac{\partial}{\partial t}\boldsymbol{q} + \frac{\partial}{\partial x}\boldsymbol{F}(\boldsymbol{q}) + \frac{\partial}{\partial y}\boldsymbol{F}(\boldsymbol{q}) = \boldsymbol{0}$$

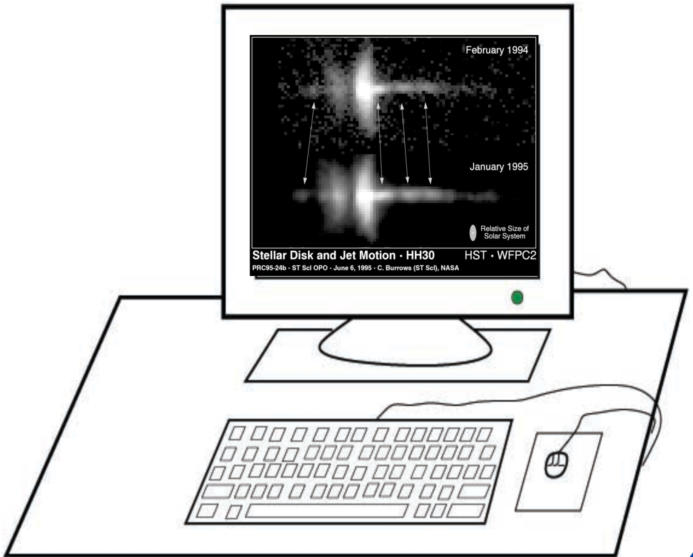
$$q_{X,i}^{n,*} = q_i^n - \frac{\Delta t}{\Delta x}\left(f\left(q_{i+\frac{1}{2}}^n\right) - f\left(q_{i-\frac{1}{2}}^n\right)\right)$$

$$q_{X+Y,i}^{n+1} = q_{X,i}^{n,*} - \frac{\Delta t}{\Delta y}\left(f\left(q_{X,i+\frac{1}{2}}^{n,*}\right) - f\left(q_{X,i-\frac{1}{2}}^{n,*}\right)\right) .$$

# Hardware

- **Machines**
  - “Deep Thought” (ref.:Douglas Adams “The Hitchhiker's Guide to the Galaxy”) or shared computing (your work/home desktop? Laptop?)
  - dedicated cluster
  - Grid technology

- **In Athens**
  - Linux machines
  - MS Windows machines
  - Cluster(s)
  - Buying NOW with REAL money: Dual Core, AMD Athlon X2, 4400, 4GB RAM, Operating System: Scientific Linux 4.2 (Kernel 2.6.9).




- **Dedicated packages or Big-General-Ultra-Mega-Giga-Hyper-code or even specially designed machine?**

# Software 1

- Codes part 1
  - ZEUS (ZEUS3D) [LCA, NCSA David Clarke, Michael Norman, Robert Fiedler] MHD 3D + Molecular
  - 3DAthena-NASA [Stephen O'Sullivan] MHD 3D
  - AMRAthena (Zeus Team) [Tom Gardiner] MHD 3D
  - AMRVAC [Rony Keppens] MHD 3D
  - AMRAstroBear [Adam Frank + Peguy Varnieres]
  - 3DMHD SPH [E. Gouveia del Pino]
  - AMR [Andrew Lim]
  - Nirvana [Udo Ziegler]
  - TD code [Turlough Downes]
  - Hydra [Turlough Downes, Stephen O'Sullivan] Multi-Fluid MHD code
  - Flash [Chicago]

# Software 2

- Codes part 2
  - Gorgon (Resistive MHD, on a fixed 3D eulerian grid,2 Temp + Q, LTE<Z> +Px), 2nd order hydro
  - Yguazu [Dublin, Raga + De Colle]
  - HyRas 3D hydro rad, cart/cyl/sph. euler, implicit solver
  - RAMSES, AMR HD multi-material gravity chemistry (CEA)(AMR + hydro + multi-Z EOS+ Chemistry)
  - Astrolabe 1D Radiative, multi-fluid, chemistry, gravity (CEA)(1D, hydro.rad, ALE multi-fluid (2 species and 2 temp.)
  - MULTI at Obspm
  - HR, 1 to 3D Eulerian cartesian (can be cylindrical or spheric),
  - grey (Rosseland) eulerian, implicit solver for R-T, gray with moments
  - Belenos (Sebastien Leygnac) 3D stationary Radiative Transfer(Exact 3D solution of radiative transfer equation) [DIAS-Cosmogrid]

# Visualisation tools

- **Visualisation AND computing**
  - IDL
  - Matlab
  - Mathematica

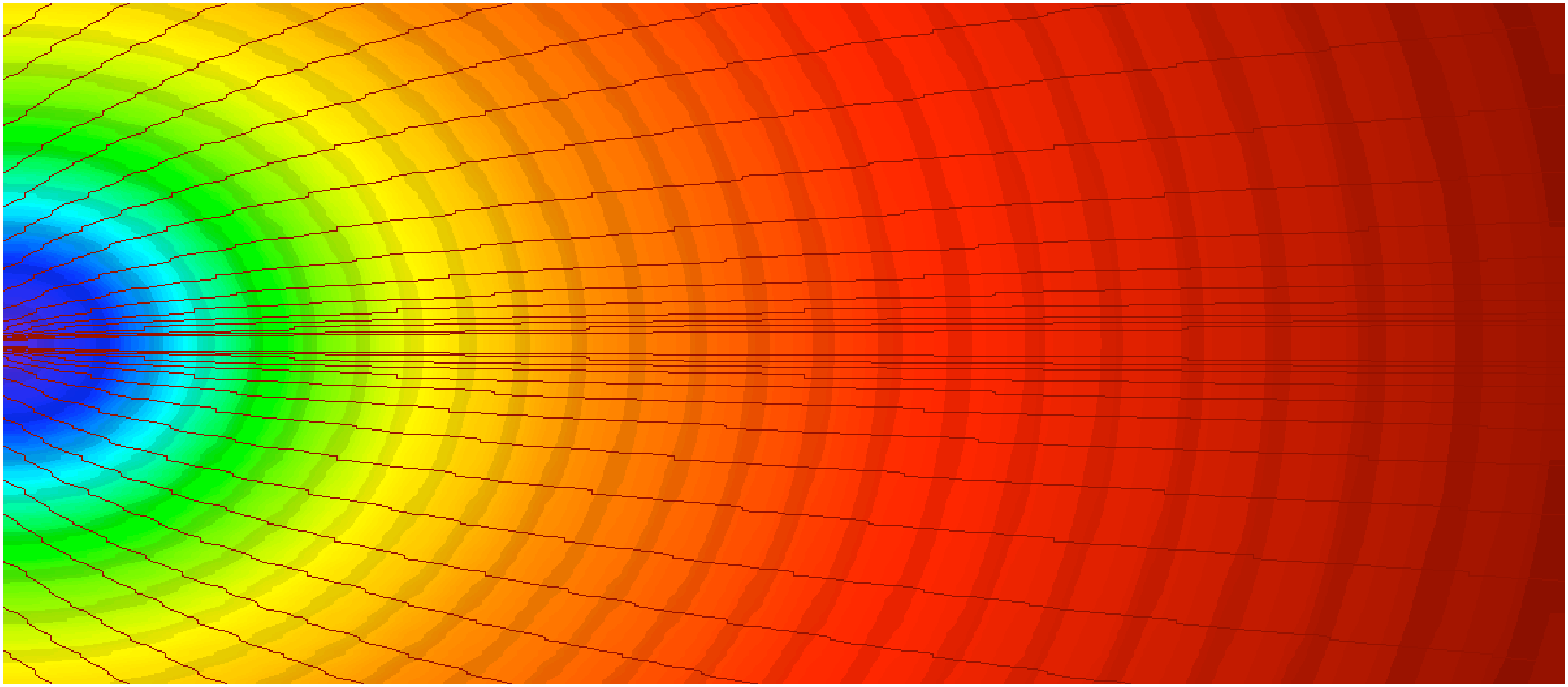
- **Visualisation only**
  - SuperMongo
  - Gnuplot
  - OpenDX

# “Movies”

- **“Movies”=animation of the simulations results**
  - animated .gifs or .MPEGs
  - propagation of the jet

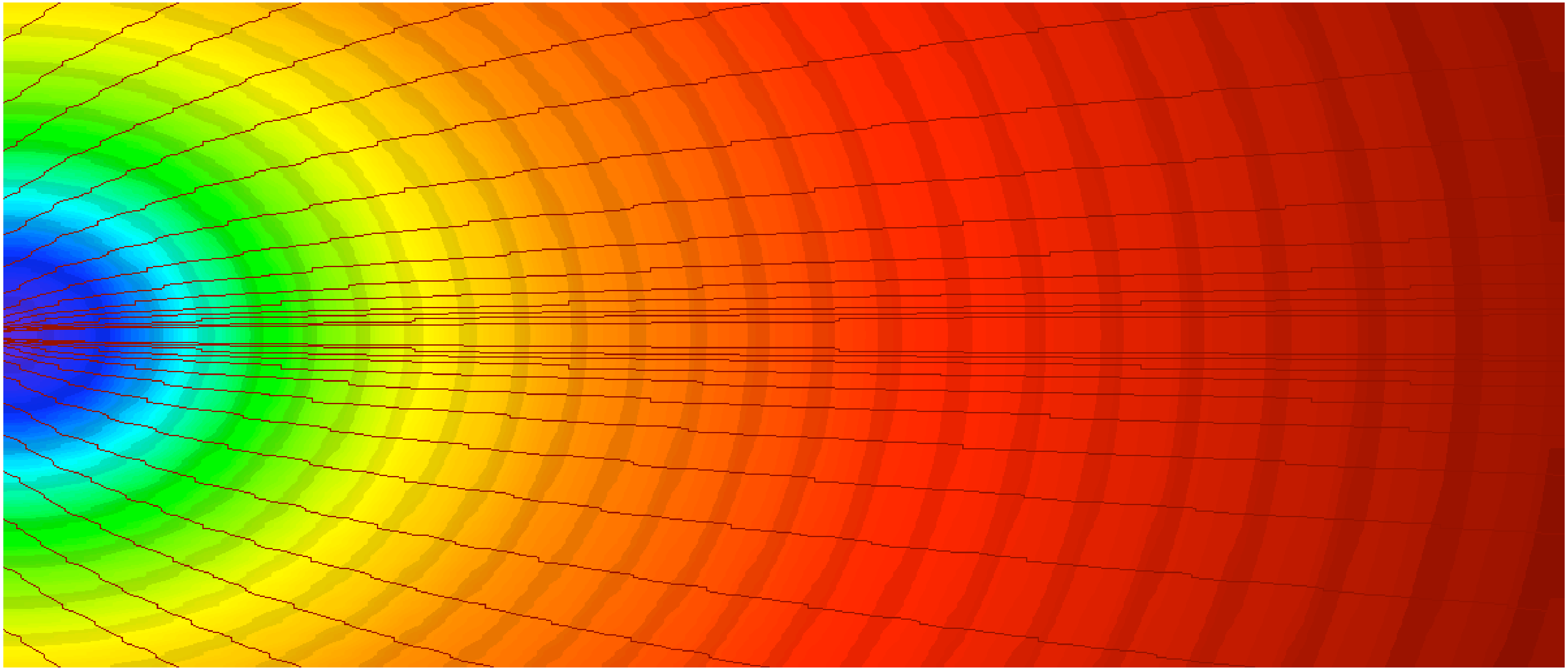
# "Movie" 1

- Numerical simulation of jet propagation



# "Movie" 2

- Is this physics or "art"?



# Prospects

- **Machines versus codes:**
  - faster machines
  - full 3D
  - larger computational boxes
  - comparisons of the results with the different codes
- **New approaches in coding, new numerical methods**
  - new numerical methods
  - “brutal force” or inovations? BOTH!

# Touch of St. Reality

- **What we can simulate today?**